Causal model for the Eating Expectations study

Brief introduction to causal models and directed acyclical graphs

Before fitting statistical models to the data, we explore a causal model for the effects of interest. The causal model is implemented as a Directed Acyclical Graph (DAG). The paths from one predictor to an outcome show the causal effect of that predictor on the outcome. Causal paths are denoted with arrows and are unidirectional, which means that causation also only goes one way. Unlike in Structural Equation Models, paths do not have to denote linear relations between variables but can for instance symbolise a quadratic relation or Bayesian inference.
We can use the “back-door criterion” to identify potential confounds of the predictors. If there is a non-causal path leading from a predictor to the outcome, a path enters the predictor. This is for instance the case if the predictor and the outcome have a common cause. Conditioning on the common cause can close this non-causal path. The aim of conditioning on variables is then to close all back-door paths so that only the causal path from the predictor to the outcome remain. If the causal assumptions in the DAG are correct, all back-door paths can be closed and the statistical model is correctly fit to the data, the observed associations can be assumed to be causal effects.

Further sources:
Glymour, M., Pearl, J., & Jewell, N. P. (2016). *Causal inference in statistics: A primer*. John Wiley & Sons.
McElreath, R. (2020). Statistical rethinking: A Bayesian course with examples in R and Stan. Chapman and Hall/CRC. 2 edition. Chapter 6
Pearl, J., & Mackenzie, D. (2018). The book of why: The new science of cause and effect. Basic books.

Directed Acyclical Graph for experiment

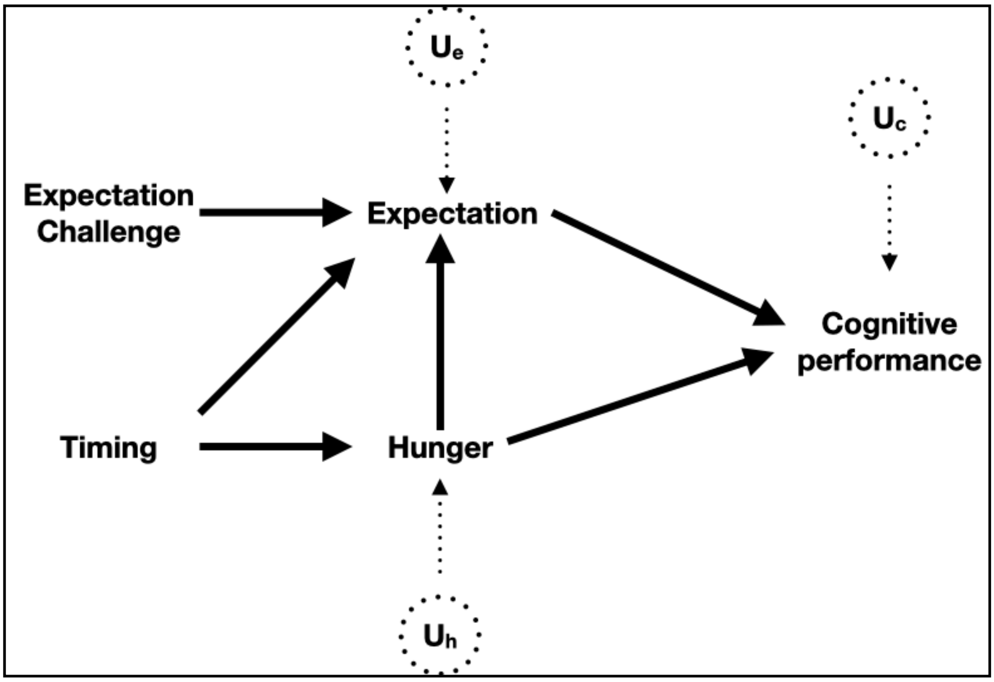

FIGURE 1. DIRECTED ACYCLICAL GRAPH
Solid arrows are causal paths of measured variables. Dotted arrows are effects of unobserved variables U.

The reasoning behind modelling the causal relations as shown in Figure 1 are based on knowledge of the research topic and the planned design of the experiment. The considerations that went into this model can be summarised as follows.

The timing of the participation in the experiment affects the hunger-level [Timing → Hunger]. The timing may also affect the expectations since participants may expect to perform worse before lunch without breakfast than after eating breakfast and lunch even before the experimental manipulation of their expectation [Timing → Expectation].

The expectation challenge affects the participant's expectations [Expectation Challenge → Expectation].

Since assignment to before/after lunch and the expectation challenge are random, there is no way in which the two predictors are connected to one another. For example, the assignment to the before-lunch condition does not affect the chance of being assigned to the "being hungry is good for concentration" expectation challenge. Hence there is no causal connection between the two interventions.

The outcome measure of interest is cognitive performance. A recent meta-analysis has shown that, on average, fasting or hungry subjects perform slightly worse than satiated subjects in a variety of cognitive performance measures (Bamberg & Moreau (unpublished). PROSPERO ID: CRD42021272822). Hence, hunger, as a sensation or

physiological state (the specific interpretation does not change the causal relations here) may affect cognitive performance [Hunger → Cognitive Performance].
The effect of expectations on cognitive performance is assumed to be influenced (as in moderated) by the subjects' hunger: the expectation may have a stronger effect on cognitive performance if the participants are more extremely hungry or satiated [Hunger → Expectation → Cognitive Performance].

There are several unmeasured variables. Cognitive performance may be influenced by things like the webbrowser used for the experiment, noise in the background, individual differences, etc.
Hunger may be influenced by the activity-level of the subjects, how much they ate and how strongly they feel hunger.
The expectations may be influenced by the participants' own beliefs on that topic.

What does the causal model imply for fitting statistical models?

The causal model can help inform decisions on including certain variables to control for confounding.
Depending on the causal effect of interest different variables may be included to ensure that all back-door paths are closed.
When looking at the effects of expectations on cognitive performance, we will also include hunger scores because this closes two back-door paths [expectations ← timing → hunger → cognitive performance, and expectation ← hunger → cognitive performance]
When considering the effector hunger on cognitive performance on the other hand, we do not condition on expectations.
There is a backdoor path from hunger to cognitive performance [hunger ← timing → expectations → cognitive performance]. However, this backdoor path is closed since the expectations are a collider [timing → expectations ← hunger] (see the above-mentioned sources for an explanation of *colliders* ("collider-bias")). Hence, this backdoor path is closed already.
Moreover, part of the effect of hunger goes through the expectations [hunger → expectations → cognitive performance]. Conditioning on expectations would close this sub-path of the causal effect of hunger. Thus, in order to find the complete effect of hunger on cognitive performance, we do not condition on expectations.
There are two *pipe-confounders* for the causal path from the interventions: hunger and expectations.
If we add hunger to a model where we are interested in effects of the timing-intervention, this effect is partly closed since it goes through hunger [timing → hunger → perf] and only the component that goes through the expectations is open.
Similarly, if we add the expectations to the equation, the effect of the expectation-intervention is closed.
If we have a statistical model with the two predictors timing-intervention and expectations, the effect of the timing intervention that goes through expectations is closed. Then, the only open causal path from the timing-intervention is through hunger.

Thus, in case the question is “what is the effect of the timing-intervention on cognitive performance that is showing itself through participants’ hunger?”, we condition on expectations.
These considerations will be taken into account when fitting the statistical models to the data.

Generative model based on the DAG:

The causal model not only helps in deciding what variables to condition on to deal with confounding. It also describes the generative process by which the outcome, cognitive performance, is generated from the factors. This generative process can be formalised and used to simulate data with.
Here, we describe the generative model used to simulate data from.

The mean cognitive performance, based on the causal relations is made up of the following factors:

$$\mu_i = \alpha_i + \beta_E * Exp_i + \beta_H * H_i.$$

(With the intercept, α_i, the expectations, Exp_i, and hunger levels H_i.)

The hunger levels are based on unobserved effects, U_H, and the effect of the intervention, $Timing_i$:

$$H_i = \beta_{U_H} U_H + \beta_{Timing[i]}.$$

In addition to unobserved effects, U_E, and the intervention, $Exp\_Challenge_i$, the expectations also depend on the hunger levels:

$$Exp_i = \beta_{U_E} * U_E + \beta_{Exp\_Challenge[i]} + \beta_{H_E} * H_i.$$

Adding this together, the model for the mean cognitive performance is:

$$\mu_i = \alpha_i + \beta_H * (\beta_{U_H} U_H + \beta_{Timing[i]}) + \beta_E * (\beta_{U_E} + \beta_{Exp\_Challenge[i]} + \beta_{H_E} * H_i).$$

Solving the parentheses:

$$y_i = \alpha_i + \beta_H * \beta_{U_H} U_H + \beta_H * \beta_{Timing[i]} + \beta_E * \beta_{U_E} * U_E + \beta_E * \beta_{Exp\_Challenge[i]} + \beta_{E\,H_E} * H_i.$$

Since there are 2*2 categorical predictors, 4 different linear equations can be written (e.g. Exp_Challenge = 0 and Timing =1).
These equations are

$$F_{(0,0)}: \ \mu_i = \alpha_i + \beta_E * \beta_{U_e} * U_E + \beta_{E\,H_E} * H_i + \beta_H * \beta_{U_h} U_H,$$

$$F_{(1,0)}: \ \mu_i = \alpha_i + \beta_H * \beta_{U_h} U_H + \beta_H * \beta_{Timing[i]=1} + \beta_E * \beta_{U_e} * U_E + \beta_{E\,H_E} * H_i,$$

$$F_{(0,1)}: \ \mu_i = \alpha_i + \beta_H * \beta_{U_h} U_H + \beta_E * \beta_{U_e} * U_E + \beta_E * \beta_{Exp\_Challenge[i]=1} + \beta_{E\,H_E} * H_i,$$

$$F_{(1,1)}: \ \mu_i = \alpha_i + \beta_H * \beta_{U_h} U_H + \beta_H * \beta_{Timing[i]=1} + \beta_E * \beta_{U_e} * U_E + \beta_E * \beta_{Exp\_Challenge[i]=1} + \beta_{E\,H_E} * H_i$$

(With F(0,0): Timing=0, Exp_Challenge=0; F(1,0): Timing=1, Exp_Challenge=0; F(0,1): Timing=0, Exp_Challenge=1; F(1,1): Timing=1, Exp_Challenge=1).

We can fit a separate linear model for each function F, so that the overall linear model of the mean likelihood is

$$\mu_i = \{ \begin{matrix} F_{(0,0)} \\ F_{(1,0)} \\ F_{(0,1)} \\ F_{(1,1)} \end{matrix} .$$

Simulation

For a simulation based on this generative model, see the R-script simulation_eating_expectation.Rmd